

Stellar models for very low mass main sequence stars: the role of model atmospheres.

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ABSTRACT

We present Very Low Mass stellar models as computed including non-grey model atmospheres for selected assumptions about the star metallicities. The role of atmospheres is discussed and the models are compared with models based on the Eddington approximation and with similar models appeared in the recent literature. Theoretical predictions concerning both the HR diagram location and the mass-luminosity relation are presented and discussed in terms of expectations in selected photometric bands. Comparison with available observational data concerning both galactic globular clusters and dwarfs in the solar neighborhood reveals a satisfactory agreement together with the existence of some residual mismatches.

Key words: stars: evolution – stars: low-mass, brown dwarfs – stars: population I and II – globular clusters: general

1 INTRODUCTION

The long-standing theoretical interest in Very Low Mass (VLM) Main Sequence stars has been recently rejuvenated according to the increasing amount of VLM objects observed with the Hubble Space Telescope as well as thanks to CCD parallax determinations (Monet et al. 1992, Dahn et al. 1995, Tinney 1996) which are increasing the amount of absolute magnitudes available for nearby dwarfs. This interest is further enhanced by the suggestion that an appreciable fraction of the baryonic mass in most galaxies could be in the form of VLM stars and Brown Dwarfs. To have light on such an observational scenario one needs reliable theoretical predictions about VLM stellar structures and, in particular, accurate mass-luminosity relations allowing the evaluation of reliable mass functions and, in turn, reliable estimates of the mass density of VLM stars in the Galaxy.

The temperature scale of M dwarfs has been for long time an unsettled problem and it is still a key ingredient for understanding the location of VLM stars in the HR diagram (Bessel 1995, Leggett et al. 1996). Since the pioneering works by Limber (1958), Hayashi & Nakano (1963) and Ezer & Cameron (1967), numerous investigators (D'Antona 1987; VandenBerg et al. 1988; Burrows, Hubbard & Lunine 1989; Dorman, Nelson & Chau 1989; Burrows et al. 1993; D'Antona & Mazzitelli 1994, 1996; Baraffe et al. 1995; Baraffe & Chabrier 1995; Baraffe & Chabrier 1996; Alexander et al. 1997; Kroupa & Tout 1997) have already devoted their attention to the structural properties and the evolutionary behavior of VLM stars. As early recognized, one knows that convection in VLM stars is very efficient throughout all the structure and that a VLM structure is very nearly adiabatic. As a consequence, theoretical predictions for the interior of such peculiar objects do not depend

on radiative opacities, nor on the choice of the mixing length parameter governing the superadiabatic convection.

In spite of these advantages, VLM stellar models critically depend on the evaluations of both opacity and equation of state (EOS) for a low temperature, high density gas, where molecules, grains and non-ideal gas effects play a relevant role. Concerning radiative opacities, large efforts have been recently devoted to properly include in opacity computations the effects of molecules and grains, with particular emphasis on the treatment of the opacity due to the H₂O and TiO molecules (see, e.g., Dorman et al. 1989, Allard & Hauschildt 1994, 1995a, 1995b, Bessel 1995). Low temperature opacity tables presented by Alexander & Ferguson (1994) and Alexander (1994) represent a significant improvement of previous evaluations given by Cox & Tabor (1976), Alexander, Johnson & Rypma (1983), and Kurucz (1992).

Substantial progresses (Saumon 1994) in the EOS for VLM stars are due to Saumon, Chabrier & Van Horn (1995) (but see also Saumon & Chabrier 1992), who provided an EOS for dense and cool matter based on a detailed description of the physics at work. However, the theory of VLM structures requires another critical ingredient, i.e., the boundary conditions for the inner stellar structure, as given by a suitable treatment of stellar atmospheres. Grey model atmospheres usually assume that the optically thin region lying above the photosphere is not affected by convection. However, a similar assumption is not well-grounded for several VLM atmospheres, since atmospheric opacities can be very large due to the presence of molecules whereas the adiabatic gradient is small due to the dissociation of H₂. As a consequence the atmosphere can be unstable against convection also at very small optical depth (τ). Moreover, non-grey effects in the stellar atmospheres are often neglected, though, theoretical evidences (Saumon et al. 1994) suggest

that such effects could play a significant role in governing the atmospheric structure.

In the last decade, significant improvements in model atmosphere for late M dwarfs have been presented by Allard (1990), Kui (1991), Brett & Plez (1993), Saumon et al. (1994), Allard & Hauschildt (1995) and Brett (1995a, 1995b). Similar models, and in particular the models provided by Saumon et al. (1994) for zero metallicity mixtures and by Allard & Hauschildt (1995) and Brett (1995a, 1995b) for finite metallicities, represent a substantial progress of our knowledge in the field. Nevertheless, Bessell (1995) has pointed out the still existing differences between different models, as due to the different evaluations of the contribution to opacity of H_2O and TiO molecules and to the different opacity averaging technique adopted in the various works.

From an evolutionary point of view, the need for accurate model atmospheres in computing VLM stellar models has been firstly stressed by Burrows et al. (1993) and more recently reinforced by Baraffe et al. (1995), Baraffe & Chabrier (1996), Chabrier, Baraffe & Plez (1996), Méra, Chabrier & Baraffe (1996). In previous works (Alexander et al. 1997, hereinafter Paper I; Brocato et al. 1997a), we have presented a theoretical approach to the evolutionary behavior of VLM stars, showing that the adoption of the most updated equation of state (Saumon et al. 1995) and low temperature opacity (Alexander & Ferguson 1994), but still relying on an approximate treatment of the stellar atmosphere, allows a rather satisfactory agreement between observation and theoretical predictions for the Color-Magnitude (CM) diagrams and the mass-luminosity relation of both metal poor and solar metallicity VLM objects. We thus concluded Paper I with the statement that “*the use of a $T(\tau)$ relation in computing stellar models has to be regarded as a first order but not-too-bad approximation to the expected evolutionary behavior*”. In this paper we will go deeper in that matter, discussing VLM stellar models including updated outer boundary conditions for various selected assumptions about the star metallicities.

The layout of this paper is as follows. Next section will provide some general informations about the models, with particular attention to the adopted grid of model atmospheres. In §3, models computed adopting model atmospheres will be compared with similar models based on the Eddington approximation and with stellar models already appeared in the literature. Section 4 presents the comparison between observational data and theoretical results. Conclusions will follow in §5.

2 BOUNDARY CONDITIONS.

Models presented in this paper adopt the same EOS and the same opacity evaluations as in Paper I. The main difference with models in Paper I is the different approach adopted for deriving the outer boundary conditions, i.e., temperature and pressure of the gas at the basis of the atmosphere. In Paper I the atmospheric integrations were performed adopting the Krishna-Swamy (1966) solar scaled $T(\tau)$ formula until reaching the optical depth $\tau = 2/3$ or, alternatively, until the onset of convection, where the standard mixing length theory was used to evaluate the degree of superadiabaticity. In the present paper outer boundary

conditions will be evaluated by adopting suitable non-grey model atmospheres.

To our knowledge, the most updated model atmospheres presently available are the ones computed by Brett (1995 a,b, hereinafter B95) and the “next generation” of the Allard & Hauschildt models (Allard & Hauschildt 1997, hereinafter NG97). Both sets of models include updated (but not identical) line lists and in both sets the numerous atomic and molecular opacity sources have been modeled with the accurate opacity sampling technique (for a detailed discussion on this point see also Bessell 1995). At the time when present computations have been performed NG97 model atmospheres were under-computing (Leggett et al. 1996, Chabrier et al. 1996) and results were available for solar metallicity only. B95 models are actually available from solar metallicity down to metallicities as low as $[M/H] \sim -2.0$. In a recent paper Chabrier et al. (1996, see also Baraffe & Chabrier 1996) have discussed the comparison between stellar models computed alternatively adopting boundary conditions from NG97, from B95, or from “old” (Base) model atmospheres by Allard & Hauschildt (1995, hereinafter AH95), emphasizing as a final result “*the excellent agreement between the observation and the stellar models based on B95 and NG97, while models based on AH95 clearly underestimate the flux in the V band*”. As pointed out by these authors, this occurrence has to be regarded as a plain evidence of the accuracy recently achieved in computing model atmospheres for M dwarfs.

In order to investigate VLM stellar structures in this paper we will adopt B95 atmospheric models, which cover the ranges $4000K \geq T_{eff} \geq 2600K$, with $\log g = 4.5, 5.0$ and for metallicities $Z = 0.0002, 0.002, 0.02, 0.04$, adopting for the mixing length $l = 1.5H_P$, where H_P is the pressure scale height. However, low metallicity VLM models reach larger effective temperature (see, e.g., Paper I) and the “critical” temperature (T_{eff}^{crit}) defining the lower limit for the validity of the Eddington approximation also increases above $4000K$ (see Baraffe et al. 1995 and reference therein). Therefore, at the lower metallicities B95 models will be implemented with Kurucz’s (1993) model atmospheres which are available for effective temperature $T_{eff} \geq 3500K$ and in a large range of gravity. One has to notice that Kurucz models lack the important H_2O opacity contribution which can be of major relevance in cool VLM objects. However, Brett (1995a; but see also Bessell 1995) has already found that, regardless of the missing H_2O opacity and the different mixing length parameter (Kurucz adopts $l = 1.25H_P$), at $4000K$ the two model atmospheres are almost identical, whereas at $T_{eff} = 3500K$ significant differences appear. Thus Kurucz models can be safely used only for temperatures above $4000K$.

When using model atmospheres as boundary conditions one has to remind that in stellar interiors the radiative flux is estimated by means of the diffusion equation which is the limit of the transfer equation for large optical depth (see Mihalas 1978 for a detailed discussion on this topic). One should thus pay attention that the assumed limit between the internal structure (where the diffusive approximation is used) and the outer atmosphere (where this approximation is no longer valid) is located at a Rosseland optical depth τ_{Ros} large enough for the diffusion approximation to be valid. Morel et al. (1994) have recently shown that for

Figure 1. The location in the HR diagram of solar metallicity, VLM stellar models computed under various assumptions (as labeled) for the boundary condition. Dotted lines connect stellar models with the same mass.

solar abundances a suitable value for τ_{Ros} is about 10. In the ATLAS9 code (Kurucz 1993), the monochromatic flux is computed through the diffusion approximation when τ_ν (i.e. the optical depth at the frequency ν) is larger or equal to 20. Thus a safe procedure is to connect the ATLAS9 model atmosphere with a model of stellar interior at an optical depth insuring the condition $\tau_\nu \geq 20$ for all frequencies, that it means $\tau_{Ros} > 20$. According to the evidence that physical mechanisms related to the energy transport in the outer layer are accounted for in a more realistic treatment in the atmosphere codes and also to fulfill the quoted condition, we fixed the bottom of the atmosphere at $\tau_{Ros} = 100$. The same value for the boundary limit has been adopted also by Baraffe & Chabrier (1996) and this occurrence should allow a safe comparison between our models and the ones provided by the quoted authors.

Nevertheless, it appears interesting to investigate how far the evolutionary behavior and in particular the location of the models in the HR diagram could be affected by different choices concerning the outer boundary limit. For this aim stellar models with solar metallicity have been computed under boundary conditions taken at different values of the Rosseland optical depth.

The computed stellar models cover the mass range $0.095M_\odot \leq M \leq 0.6M_\odot$, assuming an original Helium abundance as given by $Y=0.27$. To be consistent with evolutionary computations shown in Paper I, we have adopted for the mixing length $l = 2.2H_P$. The adopted value is different than the values adopted by Brett (1995) and Kurucz (1993) in computing their model atmospheres. However, it has been verified performing additional numerical tests that reasonable variations on the value of l have no effects at all on the determination of the stellar radius of stars less massive than about $0.5M_\odot$ (see Paper I and references therein), and only minor changes for stars with mass around $0.6M_\odot$. Since in present work, we are mainly dealing

Figure 2. The HR diagram location of 10Gyr old models for the chemical composition: $Z=0.002 - Y=0.23$, computed under different assumptions about the outer boundary conditions.

with stellar models with mass $M \leq 0.6M_\odot$, the results are not affected by the choice on the mixing length parameter. Moreover, as far as concerns the model atmosphere computations, Brett (1995) has clearly shown that a variation of the mixing length parameter in the range 1.0 to 2.0 produces only minor effects at effective temperatures around $4000K$, and that these structural effects decreases rapidly decreasing the effective temperature of the models.

Figure 1 show the HR diagram location of 10Gyr old stellar models computed adopting τ_{Ros} : 0.1, 1.0, 100. In all models the outer boundary conditions have been obtained from B95 model atmospheres alone. For the aim of comparison, the $T(\tau)$ models as given in Paper I are also presented. As expected, $T(\tau)$ models are systematically hotter and brighter than B95 models with the same mass. The maximum departure occurs at $\log T_{eff} \approx 3.52$ which corresponds to $M \approx 0.30M_\odot$, with differences $\Delta T_{eff} \approx 150K$ and $\Delta \log L/L_\odot \approx 0.08$. The origin of such behavior is related to both the H_2 dissociation mechanism and to the resulting penetration of the convection in the optical thin outer atmosphere (Auman 1969; Dorman et al. 1989; Burrows et al. 1993).

Again as expected, one finds that at smaller values of τ the location of the models is largely sensitive to the adopted value of the Rosseland optical depth where outer boundary conditions are fixed. In fact by changing τ_{Ros} from 0.1 to 1.0 the models become cooler by $\approx (70-80)K$ and fainter by $\Delta \log L/L_\odot \approx 0.05 - 0.1$. However, for $\tau_{Ros} \geq 1.0$ the models are not significantly affected by changes in the adopted value of τ_{Ros} . As a result, one finds the evidence that the evolutionary results are not dependent on the choice about the point where the atmosphere is matched with the envelope, once that a sufficiently large value ($\tau_{Ros} > 20 - 30$) is adopted in the evolutionary codes.

Bearing in mind such a scenario, stellar models for lower metallicities have been computed under the above quoted

Figure 3. As in Figure 2, but for $Z=0.0002$ - $Y=0.23$.

assumptions, for $Z=0.002$ and 0.0002 (as in B95), assuming now $Y=0.23$ and adopting outer boundary conditions either from B95 or from Kurucz's model. For each given metallicity, a set of stellar models still relying on the Eddington approximation has been also computed in order to investigate the effect of incorporating model atmospheres in stellar computations as well as to obtain indication on the value of the critical temperature T_{eff}^{crit} . Figures 2 and 3 show the results of these computations. As expected, even a quick inspection of these figures shows that at effective temperatures around $4000K$ a good match is achieved between the different models, supporting the extension of B95 with Kurucz models. More in details, when $Z=0.002$ one may notice in Figure 2 the fine smooth transition between the $T(\tau)$ stellar models and the K93 ones which occurs at $T_{eff} \approx 4400K$ for a $\approx 0.55M_{\odot}$ model. The same smooth transition occurs between K93 and B95 models at $T_{eff} \approx 4000K$ for a $\approx 0.42M_{\odot}$ object. This finding appears in fine agreement with the results by Brett (1995a, his figure 8) when comparing his model atmospheres with the ATLAS9 results. The evidence that for $T_{eff} < 4000K$ the K93 model sequence "converges" towards the $T(\tau)$ models is simply due to the missing H_2O opacity in the Kurucz's (1993) model atmospheres. The stellar models computed by adopting the Eddington approximation are hotter and brighter than B95 models. The larger discrepancy is present at $T_{eff} \approx 3800K$ where it is $\approx 140K$ in effective temperature and $\Delta \log L/L_{\odot} \approx 0.055$ in luminosity.

The HR diagram location of the stellar models with metallicity $Z=0.0002$ is displayed in Figure 3. In this case the match between the $T(\tau)$ and K93 models is achieved at $T_{eff} \approx 5000K$. This behavior appears in good agreement with result by Burrows et al (1993) concerning the increase of T_{eff}^{crit} when the heavy elements abundance is decreased. As expected, even at this very low metallicity B95 models agree with K93 ones at $T_{eff} \approx 4000K$. The maximum discrepancy between $T(\tau)$ models and stellar structures computed by adopting accurate model atmospheres appears at

$T_{eff} \approx (4100 - 4200)K$, where one finds differences of the order of $\approx 180K$ in T_{eff} and ≈ 0.003 in L/L_{\odot}

3 STELLAR MODELS.

For each given metallicity, we selected as the "best" sequence of models the one obtained by adopting in the proper range of validity the $T(\tau)$, the K93 and the B95 stellar models, paying attention that a fine and smooth match is obtained between the models computed under different assumptions about the outer boundary conditions. Figure 4 displays the run of present "best" MS in the HR diagram for the labeled assumptions about metallicity. Tables 1 through 3 give the luminosity, the effective temperature, the absolute visual magnitude and predicted colors (in the standard Johnson-Cousins system for VRI and the CIT system for K) for 10Gyr old "best" models for the various selected metallicities. Magnitudes and colors have been evaluated by adopting bolometric correction and color temperature relation from Kurucz (1993) implemented at effective temperatures lower than $4000K$ with similar evaluations given by Allard & Hauschildt (1995) or, for solar metallicity, by Allard et al. 1997 (Allard 1996), both properly shifted to overlap at each metallicity Kurucz's evaluation at the fitting point $T_{eff} = 4000K$.

The evolution of theoretical predictions concerning the HR diagram location of VLM stellar models and the mass-luminosity relation has been already discussed by Baraffe et al. (1995, hereinafter BCAH95), Baraffe & Chabrier (1996, hereinafter BC96) and Chabrier et al. (1996, hereinafter CBP96). BCAH95 presented a comparison between stellar models based either on the $T(\tau)$ or on the "old" generation of model atmospheres by Allard & Hauschildt (1995). CBP96 discussed the effect of different treatments of the atmosphere on the mass luminosity relation, presenting evolutionary computations which for the lower metallicities still rely on the AH95 model atmospheres. When revising this paper, improved models for metal-poor low-mass stars have been presented by Baraffe et al. (1997, hereinafter BCAH97) and for solar metallicity by Chabrier & Baraffe (1997, hereinafter CB97), so we have performed some additional comparisons with these updated models.

Figure 5 compares the HR diagram location of solar metallicity VLM models computed by relying either on B95 or on the Eddington approximation with models presented by BCAH95, CBP96 and CB97. One finds that present results distribute in between the "old" (BCAH95) and the "new" generation models (CBP96) presented by the group of Baraffe and coworkers. This is a rather surprising result, since present models and CB97 should have quite a similar input physics (following the results discussed by BC96), and it is not clear to us where the difference is coming from. Nor the origin of the (small) differences between BC96 and CB97 has been till now discussed in the literature. One also finds the curious evidence that most updated CB97 models appear in close agreement with our models computed with $T(\tau)$ boundary condition.

The main difference between the NG97 models (Allard & Hauschildt 1997, Leggett et al. 1996, CBP96) and the B95 ones is the adoption of different line lists for TiO and H_2O (by Jørgensen 1994 in NG97 and by Plez et al. 1992 in B95). However, BC96 (and also CBP96) compared stellar

Figure 4. The HR diagram location of 10Gyr old models for the labeled assumptions on the chemical composition.**Table 1.** Mass, luminosity, effective temperature, absolute visual magnitude and colors for stellar models with solar metallicity and $Y=0.27$, at age equal to 10Gyr.

M/M_{\odot}	$\log L/L_{\odot}$	$\log T_e$	M_V	$(V-I)$	$(V-R)$	$(V-K)$
.600	-1.152	3.590	8.954	1.965	1.057	3.817
.550	-1.313	3.570	9.656	2.192	1.147	4.180
.500	-1.468	3.554	10.276	2.352	1.220	4.450
.450	-1.616	3.541	10.806	2.455	1.273	4.635
.400	-1.751	3.531	11.263	2.531	1.311	4.771
.350	-1.881	3.524	11.674	2.587	1.339	4.866
.300	-2.002	3.518	12.056	2.639	1.365	4.953
.280	-2.061	3.516	12.230	2.657	1.373	4.983
.250	-2.159	3.511	12.553	2.707	1.397	5.066
.200	-2.360	3.501	13.254	2.841	1.457	5.285
.180	-2.459	3.495	13.652	2.944	1.502	5.452
.150	-2.638	3.482	14.420	3.152	1.600	5.809
.120	-2.887	3.459	15.721	3.571	1.813	6.557
.100	-3.147	3.429	17.462	4.213	2.175	7.737
.098	-3.181	3.425	17.704	4.301	2.229	7.905
.097	-3.200	3.422	17.868	4.365	2.268	8.029
.095	-3.239	3.419	18.084	4.431	2.309	8.155

models computed with their evolutionary code using alternatively the B95 model atmospheres and the NG97 ones, obtaining only negligible differences. As discussed by Bessel (1995, 1996), one has to remind that for temperatures cooler than 3000K B95 models have much too strong H₂O bands, indicating that the H₂O opacity is overestimated. Such occurrence should have the effect of decreasing the B95 fits of spectra of the coolest M dwarfs by about 100K (Bessel 1995), the consequences on the HR diagram location of the models being correctly valuable only when Brett model atmospheres with updated H₂O opacities will be available. However, the differences between present and CB97 models largely occur at effective temperatures larger than 3000K, thus the discrepancies can be hardly ascribed to such an effect. Here we can only conclude that such differences deserve further investigations, data in Figure 5 giving an indication

Table 2. As in Table 1, but for metallicity $Z=0.002$ and $Y=0.23$.

M/M_{\odot}	$\log L/L_{\odot}$	$\log T_e$	M_V	$(V-I)$	$(V-R)$	$(V-K)$
.700	-.514	3.717	6.273	.952	.540	1.929
.600	-.833	3.676	7.240	1.155	.661	2.381
.550	-1.030	3.643	7.945	1.379	.800	2.788
.500	-1.226	3.617	8.683	1.617	.923	3.158
.450	-1.407	3.600	9.292	1.764	.997	3.381
.420	-1.507	3.592	9.571	1.797	1.013	3.431
.400	-1.568	3.588	9.739	1.814	1.021	3.458
.380	-1.624	3.584	9.901	1.835	1.029	3.490
.350	-1.704	3.580	10.121	1.855	1.037	3.522
.320	-1.773	3.577	10.312	1.872	1.043	3.549
.300	-1.826	3.574	10.466	1.891	1.048	3.579
.250	-1.981	3.567	10.910	1.941	1.060	3.657
.200	-2.192	3.556	11.553	2.037	1.075	3.806
.180	-2.297	3.549	11.888	2.097	1.085	3.900
.150	-2.486	3.536	12.522	2.229	1.111	4.101
.120	-2.752	3.511	13.546	2.516	1.177	4.533
.110	-2.879	3.498	14.098	2.699	1.230	4.797
.100	-3.066	3.474	15.087	3.092	1.373	5.370
.099	-3.092	3.470	15.262	3.167	1.405	5.499
.098	-3.119	3.466	15.439	3.240	1.437	5.626
.096	-3.182	3.456	15.865	3.429	1.525	5.921
.095	-3.216	3.451	16.081	3.523	1.572	6.059
.093	-3.294	3.438	16.651	3.793	1.710	6.450
.092	-3.338	3.430	16.991	3.959	1.799	6.683

Table 3. As in Table 1, but for metallicity $Z=0.0002$ and $Y=0.23$.

M/M_{\odot}	$\log L/L_{\odot}$	$\log T_e$	M_V	$(V-I)$	$(V-R)$	$(V-K)$
.700	-.385	3.758	5.910	.805	.451	1.580
.600	-.743	3.712	6.883	.988	.553	1.966
.500	-1.128	3.653	8.082	1.289	.729	2.564
.450	-1.311	3.634	8.661	1.417	.807	2.785
.400	-1.463	3.622	9.132	1.506	.861	2.936
.350	-1.581	3.616	9.473	1.550	.890	3.010
.300	-1.702	3.611	9.809	1.583	.913	3.067
.250	-1.849	3.605	10.222	1.627	.940	3.138
.200	-2.056	3.595	10.813	1.698	.977	3.233
.180	-2.161	3.589	11.119	1.739	.997	3.283
.150	-2.352	3.577	11.681	1.820	1.033	3.381
.120	-2.620	3.554	12.512	1.975	1.079	3.548
.100	-2.979	3.510	13.685	2.284	1.138	3.751
.098	-3.048	3.499	13.923	2.365	1.154	3.780
.097	-3.087	3.493	14.056	2.409	1.162	3.791
.096	-3.121	3.487	14.203	2.465	1.177	3.859

of the degree of freedom still existing in theoretical predictions.

As far as the mass-luminosity relation is concerned, CBP96 have already discussed the effect of non-gray model atmospheres on the reliability of $m-L$ relation, and that discussion will not be repeated here. Moreover, Kroupa & Tout (1997) (but see also von Hippel et al. 1997) have recently presented - still relying on $T(\tau)$ stellar models - an investigation on the metallicity dependence of the theoretical mass - magnitude relation. Figure 6 shows the $m-M_V$ and $m-M_I$ relations for solar metallicity VLM objects, as obtained by using $T(\tau)$ or B95 atmospheres. For the sake of comparison we report in the same figure similar predictions from BC96, with magnitudes derived from the published luminosities according to the above quoted procedure. Figure 7 shows the mass-luminosity relations in selected photometric bands for the "best" models at the two lower metallicities

Figure 5. The location in the HR diagram of present stellar models for solar metallicity compared with similar models but from BCAH95, BC96 and CB97.

investigated in the present work. As a relevant point, one finds that the m - M_K relation appears scarcely affected by metallicity effect, at least for $M > (0.11 - 0.12)M_\odot$, i.e. for $M_K < 9.0$ mag. Such occurrence could be of some help when planning observational surveys devoted to investigate the mass function for field stars, whose metallicity is usually not well known.

In panel c) of the previous Figure, we report also the semi-empirical m - M_I relation given by Fahlman et al. (1989) for a metallicity $Z \approx 0.0001$. One finds that such an estimate appears in rather good agreement with present theoretical predictions for m - M_I at $Z=0.0002$ for $M > 0.16M_\odot$ i.e. $M_I < 10$ mag. At larger magnitudes, the semi-empirical relation crosses our predictions for $Z=0.0002$, predicting a larger magnitude for a given stellar mass. However, at this faint end of the m - M_I relation, Fahlman et al. (1989) adopted theoretical models by D’Antona (1987), computed by using ”old” physics both for the EOS and the low temperature opacity, possibly affecting the semi-empirical results for M_I magnitudes above $\approx (9 - 10)$ mag.

Since the first derivative of the mass-magnitude relation is a key tool in interpreting observed luminosity functions in term of the mass function, previous results for the various bands have been best fitted to obtain simple analytical relations of the form:

$$\log M/M_\odot = a_0 + a_1 \cdot M_x + a_2 \cdot M_x^2 + a_3 \cdot M_x^3 + a_4 \cdot M_x^4$$

Table 4 shows the values of the coefficients for the different metallicities together with the value of the standard deviation σ for each relation.

4 THEORY VERSUS OBSERVATIONS.

In Paper I we found that a rather satisfactory agreement between observational data and theoretical VLM models can

Figure 6. $m - M_V$ and $m - M_I$ relations for solar metallicity, 10Gyr old models, for various assumptions concerning the outer boundary conditions (see text).

Figure 7. As in Figure 6 but for $Z=0.002$ and $Z=0.0002$. The mass-luminosity relation is shown only for ”best” models in selected photometric bands. Panel c) shows also the m - M_I relation from Fahlman et al. (1989).

be achieved even by relying on models based on the Edington approximation. Let us here compare present ”best” models with observations.

The most relevant observational sample is obviously represented by recent Hubble Space Telescope data for lower

Table 4. Coefficients of the polynomial regression for the *mass - magnitude* relation, for the various metallicities and photometric bands adopted in the present work. The last column lists the standard deviation.

Z	M_x		a_0	a_1	a_2	a_3	a_4	σ
0.02	M_V	< 12.5mag	39.76505	-15.11174	2.13080	-0.13239	3.03803E-3	0.003
	M_V	> 12.5mag	6.22027	-1.09083	0.05592	-9.81080E-4		0.001
	M_I	< 10mag	28.51610	-13.85940	2.50028	-0.19905	5.84703E-3	0.002
	M_I	> 10mag	-14.72470	5.59494	-0.77702	4.56054E-2	-9.75915E-4	0.0005
	M_R	< 11mag	33.60587	-14.47284	2.31192	-0.16278	4.23001E-3	0.003
	M_R	> 11mag	-2.94907	1.45823	-0.22239	1.27567E-2	-2.55325E-4	0.0008
	M_K	< 7.5mag	18.32809	-11.93366	2.88362	-0.30845	1.21653E-2	0.002
	M_K	> 7.5mag	-2.70027	1.17979	-0.17640	7.51283E-3		0.0002
0.002	M_V	< 14mag	4.40273	-2.11497	0.36603	-2.77915E-2	7.49083E-4	0.013
	M_V	> 14mag	65.05768	-16.25213	1.50485	-6.21070E-2	9.62896E-4	0.0002
	M_I	< 9.0mag	8.10679	-4.42546	0.87922	-7.64219E-2	2.37850E-3	0.004
	M_I	> 9.0mag	-19.06139	7.46401	-1.07565	6.59578E-2	-1.47510E-3	0.0009
	M_R	< 9.0mag	-0.49015	0.67463	-0.22982	2.84768E-2	-1.24476E-3	0.0004
	M_R	> 9.0mag	-0.54928	0.67247	-0.14004	9.20567E-3	-2.00095E-4	0.002
	M_K	< 7.0mag	7.56689	-5.02387	1.22199	-0.13146	5.11053E-3	0.001
	M_K	> 7.0mag	0.37826	-0.05447	1.69223E-3	-3.70568E-3	2.68112E-4	0.001
0.0002	M_V	< 10mag	3.81682	-1.71337	0.26601	-1.71522E-2	3.31352E-4	0.002
	M_V	> 10mag	0.86197	3.29285E-2	-3.16077E-2	1.40673E-3		0.002
	M_I	< 8.5mag	1.97179	-0.96217	0.15045	-8.53940E-3		0.005
	M_I	> 8.5mag	-1.36246	0.68103	-0.10568	4.27400E-3		0.001
	M_R	< 8.5mag	-9.55902	5.94358	-1.37655	0.13923	-5.25802E-3	0.001
	M_R	> 8.5mag	2.08701	-0.31581	-4.86486E-3	8.30517E-4		0.002
	M_K	< 7.0mag	1.21849	-0.68787	0.12464	-9.01619E-3		0.005
	M_K	> 7.0mag	-10.15714	5.01276	-0.91221	6.91446E-2	-1.88977E-3	0.002

Figure 8. (V, V-I) CM diagram for the lower main sequence of NGC6397 (Cool et al. 1996) as compared with theoretical isochrones for $[M/H]=-2.04$, and for the ages 10, 12 and 14Gyr (Cassisi et al. 1997) shifted to account for a cluster distance modulus and reddening $(m - M)_V = 12.50$ and $E(V - I) = 0.20$. The MS locus for VLM structures, for $[M/H]=-1.04$ t=10 Gyr is also shown.

main sequences in galactic globular clusters. Several CM diagrams have been already presented (NGC6397: Paresce, De Marchi & Romaniello 1995, Cool, Piotto & King 1996 and, Mould et al. 1996; 47Tuc, M30: King, Cool & Piotto 1996,

Piotto, Cool & King 1997; NGC6752: Ferraro et al. 1997; NGC6656: De Marchi & Paresce 1997). The most tight sequence of VLM stars in a GC appears the one presented by Cool et al. (1996), which represents a fundamental tool to test the theory of VLM structures. In Paper I it has been shown that a largely satisfactory agreement has been achieved between our $T(\tau)$ models and observation. In figure 8, we perform the same comparison but using present *best* VLM models for metallicity $[M/H]=-2.04$ and -1.04 , adopting from Alcaïno et al. (1987) a cluster distance modulus $(m - M)_V = 12.50$ and a reddening $E(V - I) = 0.20$. Data in figure 8 have been implemented at larger luminosities with isochrones for the ages 10, 12 and 14Gyr (Cassisi et al. 1997), as computed by adopting the same opacity evaluations used in this work but the OPAL equation of state (Rogers, Swenson & Iglesias 1996) to allow the required match with the VLM sequence (see Brocato, Cassisi & Castellani 1997a for a discussion on that matter).

One finds that observational data agree fairly well with present theoretical predictions for metal poor models. Interesting enough, one can notice that not reasonable fitting can be achieved with the moderately metal rich sequence shown in the same figure. Thus theoretical results appear in good agreement with current estimates for the cluster metallicity, namely $[M/H] \simeq -1.61$, where an enhancement of α elements by $[\alpha/Fe] = 0.30$ has been taken into account (see Brocato, Cassisi & Castellani 1997b for more details). As shown in the same Figure, some residual discrepancies between theory and observations still exist, to be eventually better understood but only when updated theoretical color - T_{eff} relations suitable for metal poor stars will become available (Allard et al. 1997).

Figure 9 shows the most complete presently available CM diagram for stars with known parallaxes, as obtained implementing the sample provided by Monet (1992) with recent data by Dahn et al. (1995). In Paper I, it has been already shown that metal poor $T(\tau)$ models and BCAH95 models for $[M/H]=-1.5$ appear in rather good agreement with the location of the hotter subdwarfs sequence. However, that paper also disclosed that all theoretical models, including BCAH95, failed in accurately reproducing the location of fainter objects for the cooler sequence of stars, usually interpreted as the sequence of VLM stars with solar metallicity. To test if the use of more accurate outer boundary conditions can help in reducing the discrepancy between observations and theory, the same Figure 9 gives the location of "best" models, for the three metallicities adopted in this work, together with the $T(\tau)$ sequence of models for solar metallicity from Paper I and the BCAH95 and BC96 models (all for solar metallicity).

An inspection of the figure leads to the following conclusions:

i) "best" metal poor models rank very well along the hotter subdwarfs sequence, supporting the indication given in Paper I about the lower limit for the metallicity of disk subdwarfs and the evidence that the CM diagram location of VLM stars appears as a *metallicity indicator of unusual sensitivity* (Paper I, Brocato, Cassisi & Castellani 1997a, 1997b);

ii) B95 models significantly improve the fit of the metal rich sequence in comparison with *old* $T(\tau)$ models. Present models appear also in best agreement with observation with respect to BC96 models in the color range $2.2 \leq (V-I) \leq 3.0$, where the main sequence location is strongly affected by the adopted treatment for the outer boundary conditions (as discussed in Paper I and in BCAH95). Nevertheless, one finds that a significant discrepancy still exists for colors $(V-I) > 3.0$ mag, i.e. $M_V \geq 14$ mag;

iii) Curiously enough, BCAH95 models for solar metallicity seem to match the location of the cooler sequence in the CM diagram better than both present "best" and BC96 models. Since both present and BC96 models have been computed by adopting a treatment of the atmosphere much more accurate than in BCAH95, this occurrence has to be perhaps regarded as an evidence that some other "ingredient" (opacity?, color-temperature relation?), adopted in the computations, needs further improvements.

Let us finally compare our VLM stellar models with observational data recently provided by Leggett et al. (1996), who investigated 16 red dwarfs providing bolometric luminosities and effective temperatures. These authors already found a good agreement between their data and BCAH95 models. Figure 10 now compares Leggett et al. (1996) data with present "best" and BCAH97 predictions for various assumptions about the metallicity, and with BCAH95, BC96 and CB97 solar metallicity models. Inspection of the Figure shows that within the accuracy of observational data one may only conclude for a general agreement between theories and observation, without making a choice among the different approaches. In this context, it is worth noting that the object Gl 65AB which tends to be cooler than the solar metallicity sequence is an unresolved binary system. If this

Figure 9. M_V versus $(V-I)$ diagram for faint stars with known parallaxes as provided by Monet et al (1992) or Dahn et al. (1995). Theoretical predictions from this paper for the $[M/H]=0.0, -1.04$ and -2.04 , are also displayed together with the theoretical models by BCAH95, BC96 and the $T(\tau)$ models for solar metallicity from Paper I.

Figure 10. The HR diagram location of VLM object from Leggett et al. (1996) as compared with theoretical predictions from the present paper or BCAH97 for the various metallicities, as labeled. Solar metallicity models from BCAH95 and BC96 are also displayed.

system would resolved into two similar components, these would lie near the point marked by an arrow (Leggett et al. 1996), in fine agreement with the theoretical prediction.

Figure 11 compares theoretical $m-M_V$ relations from the present paper and from BCAH95 and BC96 with observational data for nearby binaries by Henry & McCarthy (1993). The two objects at $M_V \simeq 8.6$ mag (listed as Gl

Figure 11. Mass-luminosity relations for the labeled assumptions on the stellar metallicity, as obtained in this work, by BCAH95 and BC96. Observational data have been provided by Henry & McCarthy (1993).

677A and Gl 677B) which deviate significantly from theoretical prescriptions, belong to the same binary system, with no large accuracy of the orbital parameters since the orbit has been followed for less than a revolution (Henry & McCarthy 1993) Again one finds that theory and observations appear in satisfactory agreement, without allowing a choice among the different theoretical approaches.

Figure 12 finally compares the mass luminosity function given by Kroupa et al (1993) with present and previous computations on the matter (for solar metallicity), as labeled. Now one finds that the most recent results appear in better agreement, without any clear indication allowing a choice between present models and CB97. In the above quoted paper (and reference therein) Kroupa et al. discussed also the relevance of the run with M_V of the derivative of the mass-luminosity function dm/dM_V . Therefore, we have decided to compare the derivative of the mass - luminosity function obtained by adopting the most updated VLM stellar models, presently available, with the derivative of Kroupa et al.'s (1993) relation, as given by Kroupa & Tout (1997). Figure 13 shows that present models foresee a maximum in the derivative whose location appears in better agreement with observational prescriptions than CB97 models do. Thus in this respect it appears that present models work better.

5 CONCLUSIONS.

In Paper I, we investigated the effects of new physical inputs as equation of state and low temperature opacities on the evolutionary properties of VLM stellar objects, by computing stellar models relying on the Eddington approximation for the treatment of outer boundary conditions. In this work, we devoted our attention to the effect of improving the treatment of the atmosphere, by adopting Brett's (1995a,b) non-grey model atmospheres.

Figure 12. Mass - absolute visual magnitude relations for solar metallicity stellar models. Present results are plotted as solid line, models by CB97 as a dashed curve. Models of BCAH95 and BC96 are plotted as open squares and open triangles, respectively. Observational data correspond to the empirical $m - M_V$ relation derived by Kroupa, Tout & Gilmore (1993).

Figure 13. The derivative of the $m - M_V$ relation as function of M_V . The solid curve is the derivative of present models and the dashed line corresponds to the derivative obtained adopting the CB97 stellar models. The observational data correspond to the derivative of the mass - luminosity relation provided by Kroupa & Tout (1997).

Present models have been compared with other VLM stellar models in the current literature, as given by BCAH95, BC96, BCAH97 and CB97. Comparison with observational data for metal poor GC stars or solar neighborhood dwarfs shows a reasonable agreement. This result can be regarded as a plain evidence that the solution of the long-standing discrepancy between theoretical predictions and observed HR diagram location of VLM stars is no more a tantalizing goal. However, a not negligible discrepancy still exist between stellar models and CM location of the cooler sequence of VLM objects with known parallaxes, for magnitudes larger than $M_V > 14\text{mag}$. This occurrence and the

presence of still significant uncertainties in the observational data - in particular concerning the effective temperature - confirms (as pointed out by Legget et al. 1996, Bessell 1995, CBP96) that there are still improvements to be performed, both in the models (structural and atmospheric) and in the observations.

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